



Le Mans University
International Master's Degree in Electroacoustics

Maxwell-Faraday Law

Understanding Electromagnetism in Loudspeakers

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1 Introduction

This document provides a step-by-step outline of how one can connect Maxwell-Faraday's law to the familiar loudspeaker voltage equation

$$u(t) = R_e i(t) + L_e \frac{di(t)}{dt} + B\ell v(t). \quad (1)$$

1.1 Set-up, assumptions, and notation

We consider the simple configuration sketched in Fig. 1: a single closed conductive loop (a wire) representing one turn of a voice-coil. The loop bounds an oriented surface $S(t)$; its boundary is $\partial S(t)$, which is the wire itself. A two-terminal source imposes the voltage $u(t)$ across the loop terminals, and the current is denoted $i(t)$.

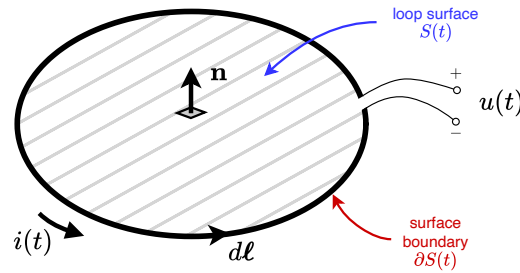


Figure 1: Single-turn loop used in the derivation. The wire (boundary $\partial S(t)$) bounds the instantaneous surface $S(t)$ with upward unit normal $\mathbf{n}$. An ideal source imposes voltage $u(t)$ and drives current $i(t)$ around the loop.

Assumptions.

- **Single turn.** For simplicity, we consider a single conductive loop. The extension to N turns (a real voice-coil) is straightforward: one simply multiplies the magnetic flux linkage (defined later) by N . The extension to multiple turns will be made explicit at the end of the derivation.
- **Motion.** The loop may translate along the vertical axis, parallel with the surface orientation vector $\mathbf{n}$, with an instantaneous conductor velocity $\mathbf{v}(t)$ (taken as positive upwards) in analogy with the motion of a loudspeaker voice-coil.
- **Instantaneous surface.** We denote by $S(t)$ the surface spanned by the loop at time t , with boundary $\partial S(t)$, as shown in Fig. 1. The explicit dependence on time emphasizes that the surface changes as the loop moves. The surface is oriented upwards, with unit normal vector $\mathbf{n}$ pointing out of the page in the upward direction. The oriented surface element is $\mathbf{n} dS$. The orientation of the boundary $\partial S(t)$ then follows from the right-hand rule with respect to $\mathbf{n}$, so that the oriented line element $d\ell$ is tangent to the loop, as indicated in Fig. 1.

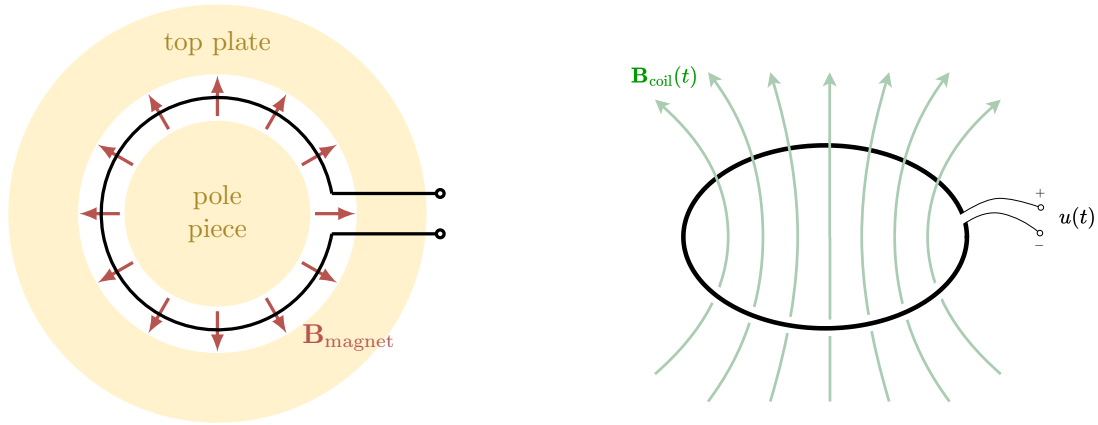
1.2 Magnetic flux and flux density.

We define the magnetic flux $\Phi(t)$ through the instantaneous surface $S(t)$ by

$$\Phi(t) = \iint_{S(t)} \mathbf{B}(t) \cdot \mathbf{n}(t) \, dA, \quad (2)$$

where dA is an infinitesimal surface element of $S(t)$, and $\mathbf{B}(t) \equiv \mathbf{B}(\mathbf{r}, t)$ is the magnetic flux density at position $\mathbf{r}$ and time t . For simplicity, we omit the explicit spatial dependence ($\mathbf{r}$) of $\mathbf{B}$ and $\mathbf{n}$ in the notation of the equations that follow.

At this stage, $\mathbf{B}$ denotes the magnetic flux density that penetrates the instantaneous surface $S(t)$ bounded by the loop. We will later discuss its physical origin: in a loudspeaker it is convenient to view $\mathbf{B}$ as the sum of two contributions, $\mathbf{B} = \mathbf{B}_{\text{magnet}} + \mathbf{B}_{\text{coil}}(t)$, where $\mathbf{B}_{\text{magnet}}$ (see Fig. 2a) is static and generated by the permanent magnets, while $\mathbf{B}_{\text{coil}}(t)$ is time-varying and generated by the current in the coil (see Fig. 2b). For the derivations that follow, only the total $\mathbf{B}$ that threads $S(t)$ matters; the split into these parts is not yet important.



(a) Top view of a single-turn wire loop in the air gap between the top plate and the pole piece of a permanent magnet. The red field lines depict only the static field $\mathbf{B}_{\text{magnet}}$ generated by the permanent magnet.

(b) A time-varying magnetic flux density $\mathbf{B}_{\text{coil}}(t)$ from the coil current.

Figure 2: Magnetic flux density contributions from the permanent magnet $\mathbf{B}_{\text{magnet}}$ (left) and the coil current $\mathbf{B}_{\text{coil}}(t)$ (right). In a loudspeaker, the total magnetic flux density $\mathbf{B}(t)$ is the sum of both contributions.

1.3 Electromotive force

The electromotive force (emf) around the moving loop is defined as

$$\mathcal{E}(t) = \oint_{\partial S(t)} \mathbf{E} \cdot d\boldsymbol{\ell} = - \frac{d\Phi(t)}{dt}. \quad (3)$$

This equation relates three quantities: the emf $\mathcal{E}(t)$, the electric field $\mathbf{E}$ along the wire loop, and the magnetic flux $\Phi(t)$ through the loop. The emf has units of volts. Physically, $\mathcal{E}(t)$ is the voltage that can be measured across the terminals of the loop and that adds to the applied source voltage $u(t)$. The middle term, $\oint_{\partial S(t)} \mathbf{E} \cdot d\boldsymbol{\ell}$, represents the circulation of the electric field $\mathbf{E}$ along the path of the wire. The last term $-\frac{d\Phi(t)}{dt}$ accounts for the change in magnetic flux through the loop.

Magnetic flux: what exactly is being integrated?

Before invoking Maxwell–Faraday’s law, it is important to emphasize the link between magnetism and electricity. Faraday’s law of induction [Eq. (3)] states that a time-varying magnetic flux $\Phi(t)$ induces an electric field $\mathbf{E}$ in the surrounding space. When integrated around a closed path, this electric field gives rise to the emf $\mathcal{E}(t)$. It is important to keep in mind that **this induced voltage exists independently of the presence of a conductor**. In other words, even if the wire loop were absent, a changing magnetic flux would still generate a *non-conservative*¹ electric field in the region. The conductive loop simply provides a physical path that allows this emf to be measured.

Without a wire loop, there are of course no terminals across which to measure a voltage. Moreover, even if one attempted to connect a voltmeter directly in such a region, the measured value would depend on the path taken by the voltmeter leads, because the induced electric field is *non-conservative*. This is precisely why the emf $\mathcal{E}(t)$ is not a potential difference in the usual sense of Kirchhoff’s voltage law, which assumes a conservative field. Instead, $\mathcal{E}(t)$ must always be understood as the line integral of the induced electric field around a specific closed contour. In our case, the natural choice of contour is the actual wire path $\partial S(t)$. Conceptually, however, the contour exists even if the conductor does not. **If a free electron were placed along this hypothetical path, it would still experience a force due to the induced electric field.**

1.4 Maxwell-Faraday Equation

The Maxwell–Faraday equation joins equations (2) and (3) and expresses how a time-varying magnetic field induces an electric field. In integral form, it is written as

$$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_S \mathbf{B} \cdot \mathbf{n} dA, \quad (4)$$

where $\mathbf{E}$ is the electric field (V/m) along the wire loop, $d\mathbf{l}$ is an infinitesimal line element along the closed contour ∂S , $\mathbf{B}$ is the magnetic flux density (T), and $\mathbf{n}dA$ is oriented surface element of surface S , bounded by ∂S .

The Maxwell–Faraday equation is fundamental to understanding loudspeaker behavior. It explains not only the voice-coil’s inductive term with inductance L_e , but also relates to electrical damping (electrical quality factor Q_{es}), and accounts for more complex effects such as eddy currents in nearby conductive parts. In what follows, we use the Maxwell–Faraday equation to derive the loudspeaker voltage equation.

2 Loudspeaker Voltage Equation

We now return to the configuration shown in Figs. 1 and 2b, which represent a single conductive loop, i.e. one turn of a loudspeaker voice-coil. Applying Kirchhoff’s voltage law (KVL) to this loop, we obtain a balance between the externally applied source voltage $u(t)$, the electrical losses in the coil wire (Ohmic losses), and any additional voltage induced by a varying magnetic flux through the loop.

Source voltage $u(t)$: We treat the source as an *ideal voltage amplifier* with zero output impedance. That is, the amplifier enforces the terminal voltage $u(t)$ independently of the current drawn by the load. Any series resistance or output impedance of a real amplifier is neglected here.

¹A non-conservative vector field is one in which the line integral between two points depends on the path taken, not just the endpoints. Contrary, in conservative fields, all paths between the same two points give the same result (so the integral around a closed loop is zero).

Ohmic losses: The resistive losses in the coil wire are modeled by the term $R_e i(t)$, where R_e denotes the electrical resistance of the coil and $i(t)$ the instantaneous current. This represents the familiar Joule heating in the conductor.

Induced voltage $\mathcal{E}(t)$: If the magnetic flux $\Phi(t)$ through the loop varies with time, Faraday's law [Eq. (3)] tells us that an induced voltage, also called electromotive force (emf), $\mathcal{E}(t)$ arises. This induced voltage acts as an additional source term in the loop equation.

Voltage balance: Combining these three contributions, KVL yields

$$u(t) + \mathcal{E}(t) = R_e i(t). \quad (5)$$

Equation (5) therefore relates the applied voltage $u(t)$, the induced emf $\mathcal{E}(t)$, and the resistive drop $R_e i(t)$ in the wire. This is the starting point for deriving the full loudspeaker voltage equation.

Starting from the relation in Eq. (5), we first move the emf to the right-hand side to isolate the applied voltage

$$u(t) = R_e i(t) - \mathcal{E}(t). \quad (6)$$

Using Faraday's law from Eq. (3), and substituting into (6) gives

$$u(t) = R_e i(t) + \frac{d\Phi(t)}{dt}. \quad (7)$$

In words: the source voltage equals the Ohmic drop in the wire plus the rate of change of magnetic flux linking the loop.

At this stage, $\Phi(t)$ denotes the total magnetic flux through the instantaneous surface $S(t)$ bounded by the loop, including both the static field of the permanent magnets and the time-varying field generated by the coil current. Rather than imposing this split a priori, the decomposition will emerge directly from the mathematical derivation.

2.1 Flux variations

Recalling the definition of magnetic flux in Eq. (2) and substituting it into the voltage balance of Eq. (7) gives

$$u(t) = R_e i(t) + \frac{d}{dt} \iint_{S(t)} \mathbf{B}(t) \cdot \mathbf{n}(t) dA. \quad (8)$$

Because both the field $\mathbf{B}(t)$ and the surface $S(t)$ can vary with time (the surface variation will be detailed later), the time derivative of the surface integral can be expanded (Leibniz product rule²) as

$$\frac{d}{dt} \iint_{S(t)} \mathbf{B} \cdot \mathbf{n} dA = \iint_{S(t)} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n} dA + \iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dA). \quad (9)$$

The two terms in Eq. (9) have distinct meanings. The first term, containing $\frac{\partial \mathbf{B}}{\partial t}$, accounts for a magnetic field that is explicitly time-varying; in a loudspeaker this is the field due to the coil current, $\mathbf{B}_{\text{coil}}(t)$. The second term treats the case where $\mathbf{B}$ itself is not explicitly time-dependent but the surface attached to the moving loop changes in time; in practice this is the motional contribution in a quasi-static gap field created by the permanent magnets, $\mathbf{B}_{\text{magnet}}$. Mathematically, this matches the intuitive split $\mathbf{B}(t) = \mathbf{B}_{\text{coil}}(t) + \mathbf{B}_{\text{magnet}}$, with the first integral

²The Leibniz product rule says that the derivative of a product of two functions equals the derivative of the first function times the second, plus the first function times the derivative of the second.

tied to the coil current contribution and the second integral to the motional contribution of a moving conductor.

Substituting the split of Eq. (9) into the voltage balance in Eq. (8) yields

$$u(t) = R_e i(t) + \underbrace{\iint_{S(t)} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n} dA}_{\text{coil current contribution}} + \underbrace{\iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t}(\mathbf{n} dA)}_{\text{motional contribution}}. \quad (10)$$

It is now tempting to define both contributions as

$$\frac{d\Phi_{\text{coil}}}{dt} = \iint_{S(t)} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n} dA, \quad (11)$$

$$\frac{d\Phi_{\text{magnet}}}{dt} = \iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t}(\mathbf{n} dA), \quad (12)$$

and substitute them into (10), to obtain

$$u(t) = R_e i(t) + \frac{d\Phi_{\text{coil}}}{dt} + \frac{d\Phi_{\text{magnet}}}{dt}. \quad (13)$$

While convenient, the notation $\frac{d\Phi_{\text{magnet}}}{dt}$ can be conceptually misleading: it seems to suggest that the magnet's own flux is changing in time. In reality, the permanent-magnet field is (to first order) static; the variation arises because the coil moves and thereby samples different parts of an otherwise steady field. It is important to keep in mind that $\Phi(t)$ is not a physical quantity attached to a fixed place in space, but the flux *seen* (linked) by the coil; if the coil moves, $\Phi(t)$ can change solely due to that motion. For this reason, in what follows we will avoid the shorthand $\frac{d\Phi_{\text{magnet}}}{dt}$ and retain its integral form $\iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t}(\mathbf{n} dA)$. By contrast, $\frac{d\Phi_{\text{coil}}}{dt}$ is self-explanatory: it is the change of flux created by the coil itself due to the alternating current flowing in the winding.

Collecting the terms in the order requested, the final loop-voltage expression is

$$u(t) = R_e i(t) + \frac{d\Phi_{\text{coil}}(t)}{dt} + \iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t}(\mathbf{n} dA), \quad (14)$$

where the three contributions are, respectively: Joule losses in the wire, the induced voltage from the coil's own time-varying flux, and the contribution from the coil's movement in the permanent-magnet field.

2.2 Coil current contribution

A current $i(t)$ running in a wire creates a magnetic field $\mathbf{B}_{\text{coil}}(t)$ around the wire (right-hand rule). For a single closed loop, the total magnetic flux $\Phi_{\text{coil}}(t)$ linked by the loop is obtained using Eq. (2) by integrating the magnetic flux density $\mathbf{B}$ through the instantaneous surface $S(t)$ spanned by the loop. The inductance L_e of such a loop is defined as the magnetic flux produced per unit current; it measures how much flux is linked by the loop for a given current. For a single loop this gives

$$\Phi_{\text{coil}}(t) = L_e i(t) \quad (15)$$

Taking the time derivative of (15) (assuming L_e does not depend on time) yields

$$\frac{d\Phi_{\text{coil}}(t)}{dt} = L_e \frac{di(t)}{dt}. \quad (16)$$

Substituting this result into Eq. (14) gives

$$u(t) = R_e i(t) + L_e \frac{di(t)}{dt} + \iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t}(\mathbf{n} dA), \quad (17)$$

2.3 Motional contribution

In this subsection we focus on the last term of Eq. (17), which captures the voltage induced by motion of the loop in a magnetic field. The integrand contains the factor $\frac{\partial}{\partial t}(\mathbf{n} d\mathbf{A})$, which is the time derivative of the oriented surface element. This can be unintuitive at first glance: “change of surface” might suggest a deformation or a change in size/shape. However, even if its size and shape are kept identical, translating this surface from a region with some magnetic field to a region with another magnetic field may modify the flux that threads it. Thus, a pure rigid motion of the same surface can change the flux seen by the coil.

To make this easier to understand, it is convenient to discretize time and replace dt by a small increment Δt . Then the loop boundary is $\partial S(t)$ at time t , and $\partial S(t + \Delta t)$ at time $t + \Delta t$. With this, we approximate the time derivative of the oriented surface element by a finite difference of surfaces attached to the successive boundaries

$$\iint_{S(t)} \frac{\partial}{\partial t}(\mathbf{n} d\mathbf{A}) \approx \frac{\Delta \mathbf{S}}{\Delta t} \equiv \frac{\mathbf{S}(t + \Delta t) - \mathbf{S}(t)}{\Delta t}. \quad (18)$$

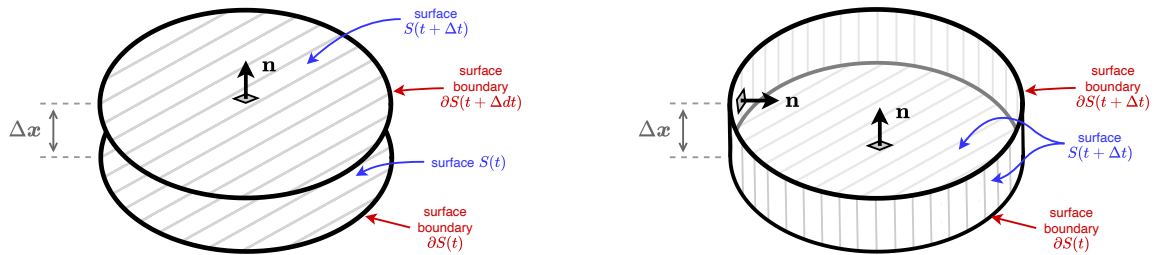
This viewpoint will let us visualize and compute the effect of boundary motion on the flux.

Fig. 3a illustrates the surface boundary $\partial S(t)$ with its attached surface $\mathbf{S}(t)$ at time t , and the same boundary displaced upward by Δx at time $t + \Delta t$, giving $\partial S(t + \Delta t)$ with an attached surface $\mathbf{S}(t + \Delta t)$. A natural but not always insightful choice is to take $\mathbf{S}(t + \Delta t)$ as the “smallest” surface spanned by the new boundary (minimal cap), as shown in Fig. 3a. While intuitive, this choice makes it harder to see explicitly how the motion relates to the difference $\mathbf{S}(t + \Delta t) - \mathbf{S}(t)$.

Fig. 3b shows a more instructive construction: the original surface $\mathbf{S}(t)$ with boundary $\partial S(t)$ remains as in Fig. 3a, and the new boundary $\partial \mathbf{S}(t + \Delta t)$ is placed at the displaced position as before; however, the attached surface $\mathbf{S}(t + \Delta t)$ is now chosen to overlap $\mathbf{S}(t)$. Recall that Eq. (4) (repeated here)

$$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_S \mathbf{B} \cdot \mathbf{n} dA,$$

specifies the boundary ∂S on the left-hand side, while on the right-hand side S may be any surface bounded by that same ∂S . Because magnetic flux density satisfies $\text{div } \mathbf{B} = 0$ (no sources/sinks), field lines that enter through the boundary must exit elsewhere; hence, for a given boundary, the total flux through any attached surface is the same. This freedom lets us choose such surfaces for which the difference $\mathbf{S}(t + \Delta t) - \mathbf{S}(t)$ is easier to visualize and calculate.



(a) The two selected surfaces are disjoint and share no common area.

(b) The two selected surfaces share a common area. The surface $\mathbf{S}(t)$ and its boundary $\partial S(t)$ remains unchanged relative to Fig. 3a; the surface $\mathbf{S}(t + \Delta t)$, attached to the same boundary $\partial \mathbf{S}(t + \Delta t)$ as in Fig. 3a, is now chosen to overlap $\mathbf{S}(t)$.

Figure 3: Schematic representation of the moving surface $\mathbf{S}(t)$ bounded by the loop $\partial S(t)$. The surface moves from position $\mathbf{S}(t)$ at time t to position $\mathbf{S}(t + \Delta t)$ at time $t + \Delta t$, traversing a distance Δx .

In Fig. 3b, think of $\mathbf{S}(t + \Delta t)$ as the “open” top of a Camembert cheese box whose rim is the boundary $\partial S(t + \Delta t)$. The bottom of this box corresponds to $\mathbf{S}(t)$. The finite difference $\mathbf{S}(t + \Delta t) - \mathbf{S}(t)$ then isolates the side “wall” of the box. What remains is the thin cylindrical shell swept by the moving boundary depicted in Fig. 4. This shell has height Δx and is oriented with inward normal $\mathbf{n}$.

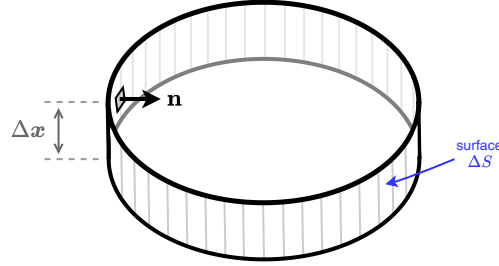


Figure 4: Thin cylindrical shell swept by the moving boundary. The shell has height Δx and is oriented with inward normal $\mathbf{n}$. The flux through this shell is the difference $\mathbf{S}(t + \Delta t) - \mathbf{S}(t)$.

To finally calculate the surface integral from the last term of Eq. (17), we need to evaluate the rate of change of flux through this thin cylindrical shell. To do so, we zoom in on a small element of this shell, as shown in Fig. 5.

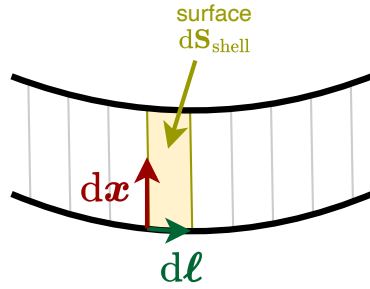


Figure 5: Zoom on a small element of the cylindrical shell swept by the moving boundary. The shell element has area vector $d\mathbf{S}_{\text{shell}} = d\mathbf{x} \times d\boldsymbol{\ell}$.

We pick a small element of the bottom boundary ($\partial S(t)$) of length $d\boldsymbol{\ell}$ (pointing to the right); let the boundary velocity be the vector $\mathbf{v}$, pointing upwards. During the short interval dt , the boundary translates by $d\mathbf{x} = \mathbf{v} dt$. The area vector of the corresponding shell patch is given by the geometric identity

$$d\mathbf{S}_{\text{shell}} = d\mathbf{x} \times d\boldsymbol{\ell}, \quad (19)$$

and points inward, parallel to the normal $\mathbf{n}$ of the original surface $S(t)$ (Fig. 4).

The total surface integral over the entire shell is obtained by integrating this area element around the entire boundary $\partial S(t)$, i.e. along the entire wire loop. This gives

$$\iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dA) = \iint_{\Delta S} \mathbf{B} \cdot \frac{d\mathbf{S}_{\text{shell}}}{dt} = \oint_{\partial S(t)} \mathbf{B} \cdot \frac{d\mathbf{x} \times d\boldsymbol{\ell}}{dt} = \oint_{\partial S(t)} \mathbf{B} \cdot \mathbf{v} \times d\boldsymbol{\ell}. \quad (20)$$

Using the scalar triple product identity, $\mathbf{B} \cdot (\mathbf{v} \times d\boldsymbol{\ell}) = (\mathbf{B} \times \mathbf{v}) \cdot d\boldsymbol{\ell}$, the motional term takes the familiar “motional emf” form

$$\iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dS) = \oint_{\partial S(t)} (\mathbf{v} \times \mathbf{B}) \cdot d\boldsymbol{\ell}. \quad (21)$$

Under the common condition that $\mathbf{v}$, $\mathbf{B}$, and the wire tangent $d\boldsymbol{\ell}$ are mutually perpendicular everywhere along the loop (as in a typical loudspeaker gap), the line integral then reduces to

$$\oint_{\partial S(t)} (\mathbf{v} \times \mathbf{B}) \cdot d\boldsymbol{\ell} = B\ell v(t), \quad (22)$$

where ℓ is the total length of the loop, i.e., the length of the coil wire.

As promised at the assumptions of section 1.1 — where we began with a single-turn winding — we now discuss how the extension to multiple turns enters. Indeed, the explicit number of turns does not appear separately: the line integral runs along the entire wire path, so what matters is the total contour length. If the coil has N turns, the contour simply wraps around N times, which is equivalently accounted for by the total wire length $\ell = N \ell_{\text{turn}}$. The effect of multiple turns is already included via ℓ .

2.4 Final loudspeaker voltage equation

Substituting the motional term from Eq. (22) into Eq. (17) gives the final loudspeaker voltage equation

$$u(t) = R_e i(t) + L_e \frac{di(t)}{dt} + B\ell v(t). \quad (23)$$

This equation relates the applied voltage $u(t)$ to the current $i(t)$ in the coil and its velocity $v(t)$ in the magnetic field. The three terms on the right-hand side represent, respectively, the Ohmic losses $R_e i(t)$ in the coil wire, the inductive voltage $L_e \frac{di(t)}{dt}$ due to the coil's own time-varying flux (self-inductance), and the motional voltage $B\ell v(t)$ induced by the coil's movement in the permanent-magnet field.

3 Discussion

It is important to keep in mind that the Maxwell–Faraday equation provides the common physical origin of the two “electrical” terms in the loudspeaker voltage law. The inductive voltage $L_e \frac{di(t)}{dt}$ (self-inductance) and the motional voltage $B\ell v(t)$ both arise from the same phenomenon: a change of magnetic flux linked by the wire. In the first case, $d\Phi/dt$ is produced because the coil's own current makes $\mathbf{B}_{\text{coil}}(t)$ explicitly time-dependent; in the second, $d\Phi/dt$ appears because the conductor moves through a static magnetic field $\mathbf{B}_{\text{magnet}}$ of the permanent magnet. In both situations, the measured voltage contribution is nothing more than the rate of change of linked flux dictated by Maxwell–Faraday, and the split into “inductive” and “motional” terms is simply a matter of convenience and interpretation.

Finally, note that eddy currents in nearby conductive parts also follow directly from Maxwell–Faraday. It is important to keep in mind, that whenever the magnetic flux changes in time, it generates a non-conservative electric field in space; inside a conductor this field finds closed paths and drives circulating currents. Those currents create their own opposing magnetic field (Lenz's law) and modify the total field seen by the coil. In loudspeakers this occurs in pole pieces, top plates, and intentionally in shorting rings or caps.