

Maxwell-Faraday equation in loudspeakers

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Goal

Connect Maxwell-Faraday's law

$$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_S \mathbf{B} \cdot \mathbf{n} dA,$$

to the familiar loudspeaker voltage equation

$$u(t) = R_e i(t) + L_e \frac{di(t)}{dt} + B\ell v(t)$$



1 Set-up, assumptions, and notation

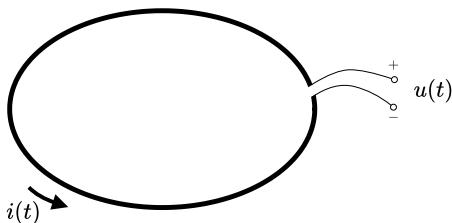
Kirchhoff Voltage Law (KVL) and emf

Maxwell-Faraday equation

Single closed conductive loop (one turn of a voice-coil)

$u(t)$ - applied voltage
(supplied by an ideal amplifier)

$i(t)$ - the resulting current
through the coil



Single closed conductive loop (one turn of a voice-coil)

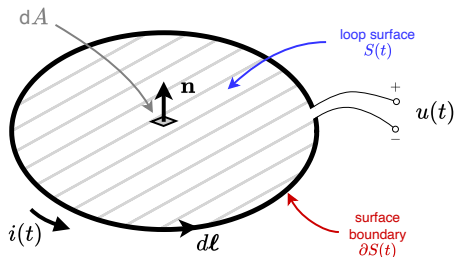
Boundary ∂S
(defined by the loop)

∂S bounds an oriented **surface $S(t)$**
with unit normal $\mathbf{n}$
(our choice)

dA is an infinitesimal piece
of surface area $S(t)$

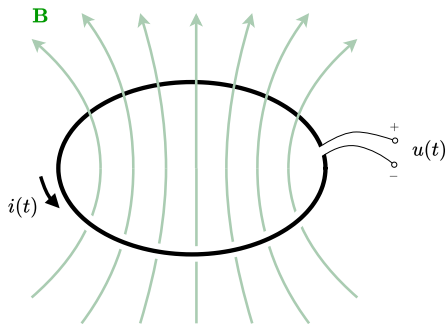
$d\ell$ is an infinitesimal element of boundary ∂S

Loop may translate vertically with velocity $\mathbf{v}(t)$



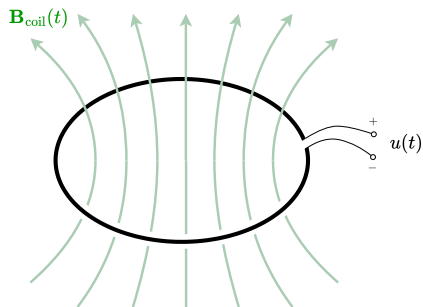
Magnetic flux through the loop

$$\Phi = \iint_S \mathbf{B} \cdot \mathbf{n} dA$$

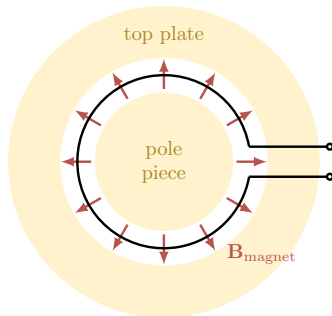


Magnetic field (flux density) B

Time-varying field $B_{\text{coil}}(t)$
due to the current in the coil



Static field B_{magnet}
due to the permanent magnet





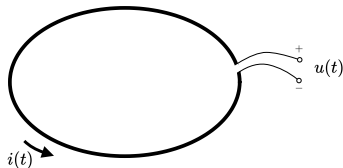
Set-up, assumptions, and notation

2 Kirchhoff Voltage Law (KVL) and emf

Maxwell-Faraday equation

Kirchhoff Voltage Law (KVL)

The directed sum of the potential differences (voltages) around any closed loop is zero.

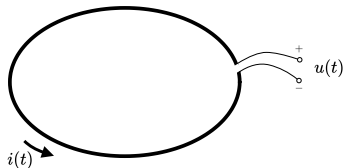


$$u(t) = R_e i(t)$$

↖ ohmic losses

Kirchhoff Voltage Law (KVL) $\rightarrow$ Faraday's law

The directed sum of the potential differences (voltages) around any closed loop is zero.



$$\mathcal{E}(t) + u(t) = R_e i(t)$$

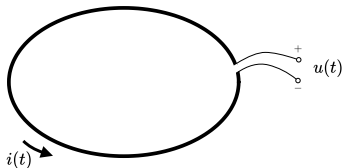
induced emf

(KVL does not hold)

ohmic losses

Kirchhoff Voltage Law (KVL) → Faraday's law

The directed sum of the potential differences (voltages) around any closed loop is zero.



$$\mathcal{E}(t) + u(t) = R_e i(t)$$

induced emf $\nearrow$ (KVL does not hold) $\nwarrow$ ohmic losses

$$\mathcal{E}(t) = -\frac{d\Phi(t)}{dt}$$

Kirchhoff Voltage Law (KVL) with Faraday's law

$$\mathcal{E}(t) + u(t) = R_e i(t)$$

$$u(t) = R_e i(t) - \mathcal{E}(t) = R_e i(t) + \frac{d\Phi(t)}{dt}$$

Source voltage equals Ohmic drop plus rate of change of linked flux.

Let's solve the flux variation term $\frac{d\Phi(t)}{dt}$.



3

Set-up, assumptions, and notation

Kirchhoff Voltage Law (KVL) and emf

Maxwell-Faraday equation

Maxwell-Faraday equation

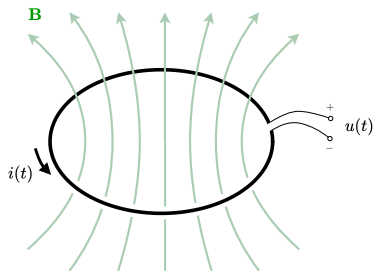
$$\mathcal{E}(t) = - \frac{d\Phi(t)}{dt}$$

The voltage emf is the total
electric field accumulated
along the closed contour

total magnetic flux
through the surface S
and its rate of change

$$\oint_{\partial S} \mathbf{E} \cdot d\boldsymbol{\ell} = - \frac{d}{dt} \iint_S \mathbf{B} \cdot \mathbf{n} dA$$

Kirchhoff Voltage Law (KVL) with Faraday's law



$$u(t) = R_e i(t) + \frac{d}{dt} \iint_S \mathbf{B} \cdot \mathbf{n} dA$$

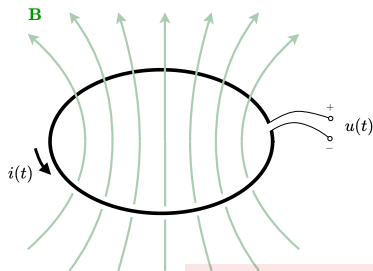
ohmic losses



Maxwell-Faraday term



Kirchhoff Voltage Law (KVL) with Faraday's law



$$u(t) = R_e i(t) + \frac{d}{dt} \iint_S \mathbf{B} \cdot \mathbf{n} dA$$

ohmic losses



Maxwell-Faraday term



Both the field $\mathbf{B}(t)$ and the surface $S(t)$ can vary with time

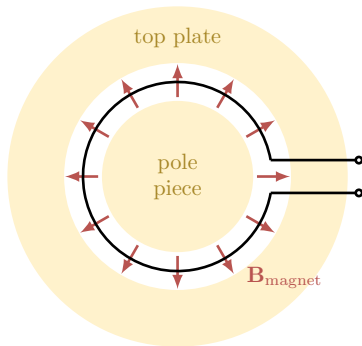
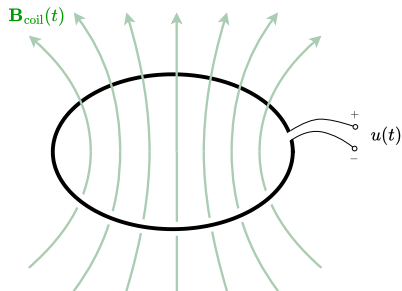
$$\frac{d}{dt} \iint_{S(t)} \mathbf{B} \cdot \mathbf{n} \, dA = \iint_{S(t)} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n} \, dA + \iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} \, dA)$$

product rule for differentiation

$$\frac{d}{dt} (f(t) \cdot g(t)) = f'(t) \cdot g(t) + f(t) \cdot g'(t)$$

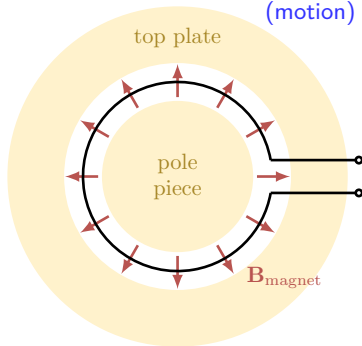
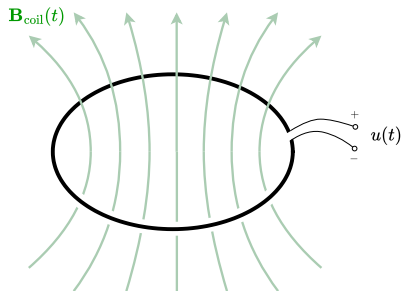
Both the field $\mathbf{B}(t)$ and the surface $S(t)$ can vary with time

$$\frac{d}{dt} \iint_{S(t)} \mathbf{B} \cdot \mathbf{n} dA = \iint_{S(t)} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n} dA + \iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dA)$$



Both the field $\mathbf{B}(t)$ and the surface $S(t)$ can vary with time

$$\frac{d}{dt} \iint_{S(t)} \mathbf{B} \cdot \mathbf{n} dA = \iint_{S(t)} \underbrace{\frac{\partial \mathbf{B}}{\partial t}}_{\text{change of } \mathbf{B}} \cdot \mathbf{n} dA + \iint_{S(t)} \mathbf{B} \cdot \underbrace{\frac{\partial}{\partial t}(\mathbf{n} dA)}_{\text{change of surface (motion)}}$$



$$u(t) = R_e i(t) + \underbrace{\iint_{S(t)} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n} \, dA}_{\text{coil current contribution}} + \underbrace{\iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} \, dA)}_{\text{motional contribution}}$$

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$\frac{d\Phi_{\text{coil}}}{dt}$... flux variation due to the coil current

Coil inductance

Inductance $L_e \equiv$ the ratio of magnetic flux to current

it measures how much flux is linked by the loop for a given current.

For a single loop this gives:

$$\Phi_{\text{coil}}(t) = L_e i(t)$$

$$\frac{d\Phi_{\text{coil}}(t)}{dt} = L_e \frac{di(t)}{dt}$$

assuming L_e does not depend on time

$$u(t) = R_e i(t) + \underbrace{\iint_{S(t)} \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n} \, dA}_{\text{coil current contribution}} + \underbrace{\iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} \, dA)}_{\text{motional contribution}}$$

$$u(t) = R_e i(t) + L_e \frac{di(t)}{dt} + \iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} \, dA)$$

The coil current contribution of the Maxwell-Faraday equation results in the familiar inductance term

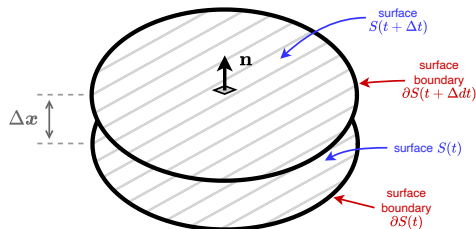
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$$u(t) = R_e i(t) + L_e \frac{di(t)}{dt} + \iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} \, dA)$$

voltage induced by motion of the loop in a magnetic field

Let's discretize and simplify for better understanding

$$\iint_S \frac{\partial}{\partial t} (\mathbf{n} \, dA) \approx \frac{\Delta \mathbf{S}}{\Delta t} \equiv \frac{\mathbf{S}(t + \Delta t) - \mathbf{S}(t)}{\Delta t}$$

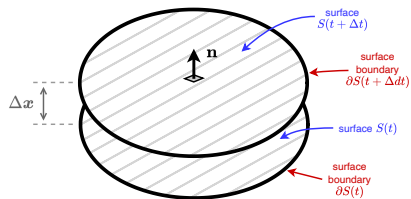


Choice of attached surface

$$\oint_{\partial S} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_S \mathbf{B} \cdot \mathbf{n} dA$$

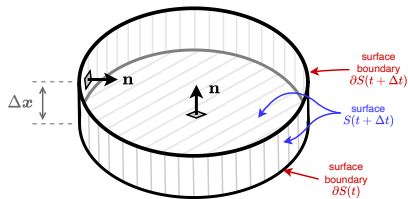
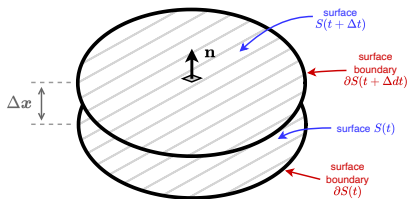

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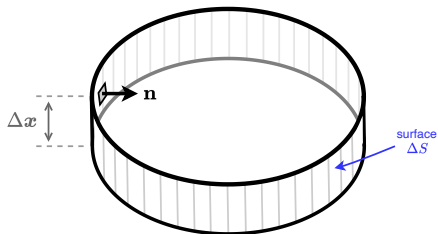
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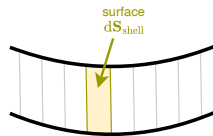
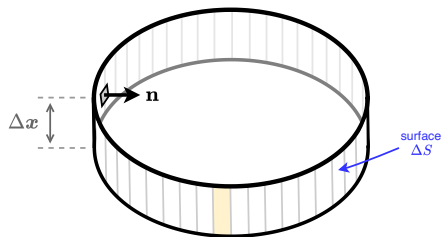


Surface differentiation

$$\iint_S \frac{\partial}{\partial t} (\mathbf{n} \, dA) \approx \frac{\Delta \mathbf{S}}{\Delta t} \equiv \frac{\mathbf{S}(t + \Delta t) - \mathbf{S}(t)}{\Delta t}$$

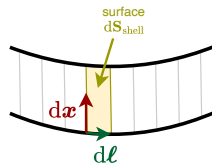
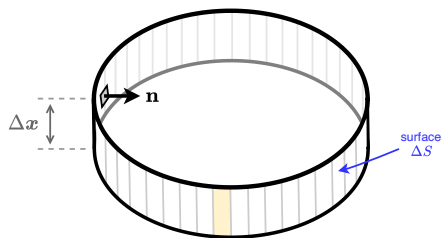


Surface differentiation



$$\iint_S \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dA) = \iint_{\Delta S} \mathbf{B} \cdot \frac{d\mathbf{S}_{\text{shell}}}{dt}$$

Surface differentiation

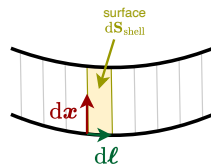
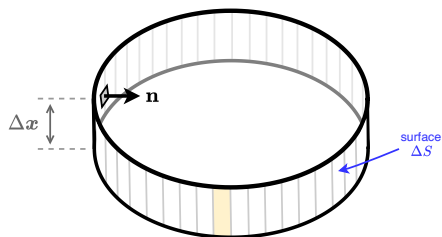


$$d\mathbf{S}_{\text{shell}} = dx d\ell \mathbf{n}$$

$$d\mathbf{S}_{\text{shell}} = d\mathbf{x} \times d\ell$$

$$\iiint_S \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dA) = \iiint_{\Delta S} \mathbf{B} \cdot \frac{d\mathbf{S}_{\text{shell}}}{dt}$$

Surface differentiation

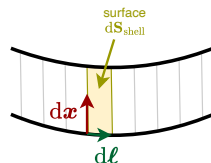
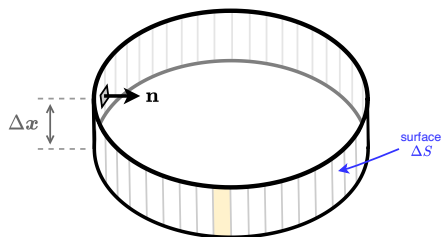


$$d\mathbf{S}_{\text{shell}} = dx d\ell \mathbf{n}$$

$$d\mathbf{S}_{\text{shell}} = d\mathbf{x} \times d\mathbf{\ell}$$

$$\iint_S \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dA) = \iint_{\Delta S} \mathbf{B} \cdot \frac{d\mathbf{S}_{\text{shell}}}{dt} = \oint_{\partial S(t)} \mathbf{B} \cdot \frac{d\mathbf{x} \times d\mathbf{\ell}}{dt}$$

Surface differentiation



$$d\mathbf{S}_{\text{shell}} = dx d\ell \mathbf{n}$$

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$$\iiint_S \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dA) = \iint_{\Delta S} \mathbf{B} \cdot \frac{d\mathbf{S}_{\text{shell}}}{dt} = \oint_{\partial S(t)} \mathbf{B} \cdot \frac{d\mathbf{x} \times d\mathbf{\ell}}{dt} = \oint_{\partial S(t)} \mathbf{B} \cdot \mathbf{v} \times d\mathbf{\ell}$$

Final Equation

If $\mathbf{v} \perp \mathbf{B} \perp d\boldsymbol{\ell}$ along the loop of length ℓ

$$\iint_{S(t)} \mathbf{B} \cdot \frac{\partial}{\partial t} (\mathbf{n} dA) = \oint_{\partial S(t)} \mathbf{B} \cdot \mathbf{v} \times d\boldsymbol{\ell} = \oint_{\partial S(t)} (\mathbf{v} \times \mathbf{B}) \cdot d\boldsymbol{\ell} = B\ell v(t)$$

$$u(t) = R_e i(t) + L_e \underbrace{\frac{di(t)}{dt}}_{\text{both terms come from Maxwell-Faraday equation}} + B\ell v(t)$$

both terms come from
Maxwell-Faraday equation