

Synchronized Swept-Sine

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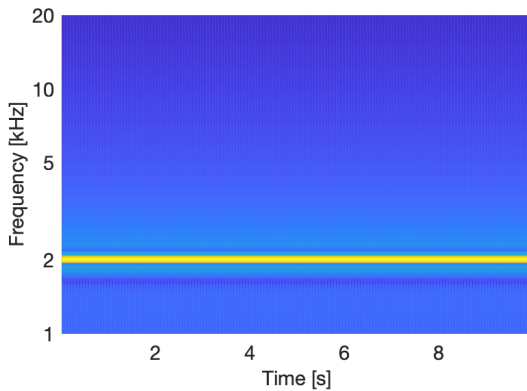
... a synchronous swept-sine ...

a signal that can be easily used for
analysis of nonlinear systems

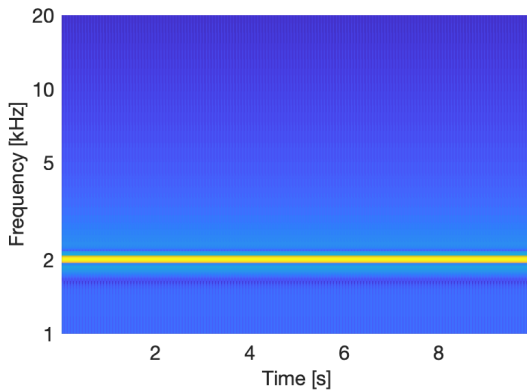
spectrogram quiz

level 1

Question: what it sounds like ?

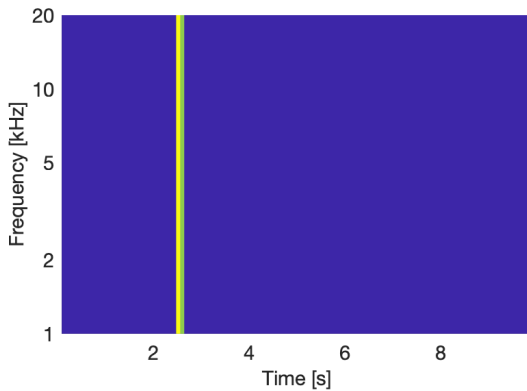


Question: what it sounds like ?

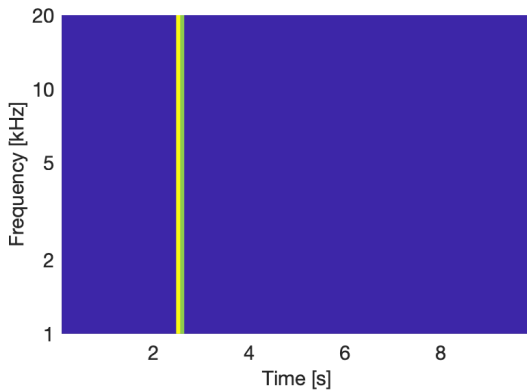


Answer: sine wave

Question: what it sounds like ?

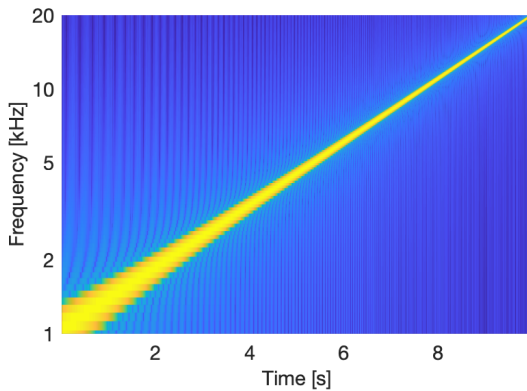


Question: what it sounds like ?

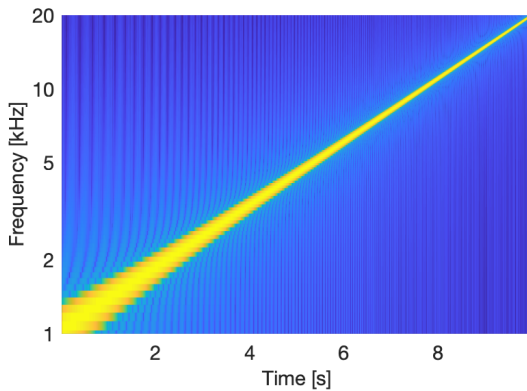


Answer: Dirac impulse (or an impulsive sound)

Question: what it sounds like ?



Question: what it sounds like ?

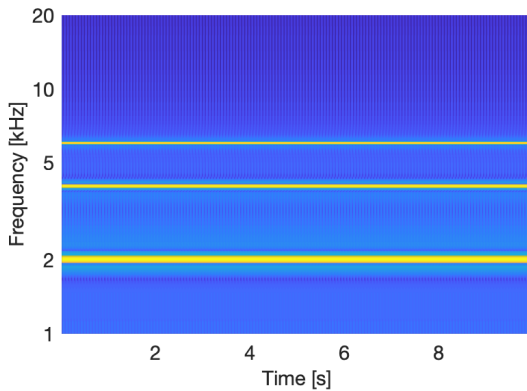


Answer: swept-sine

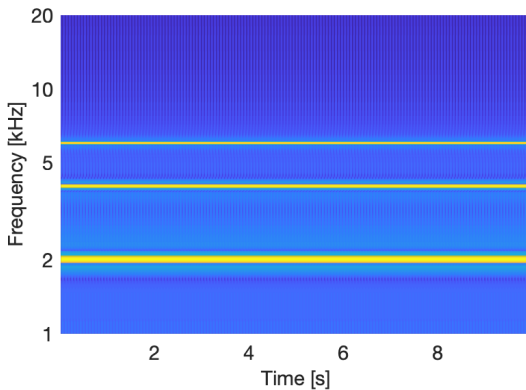
spectrogram quiz

level 2

Question: what it sounds like ?



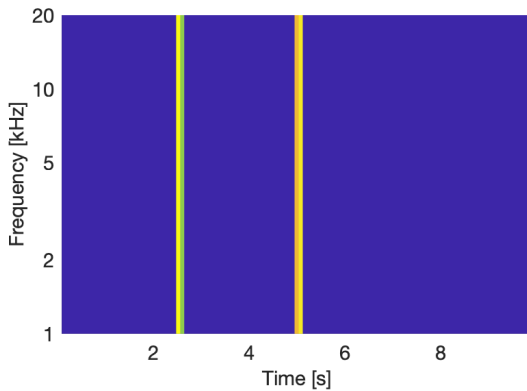
Question: what it sounds like ?



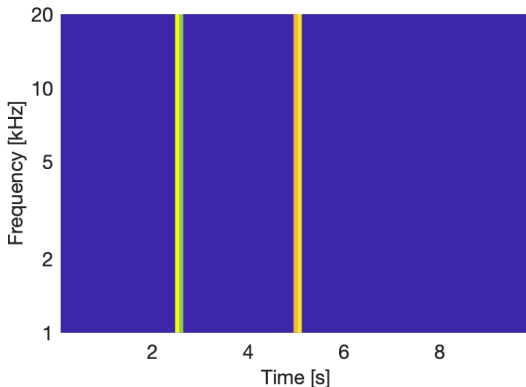
Answer 1: three independent tone signals

Answer 2: distorted sine wave

Question: what it sounds like ?



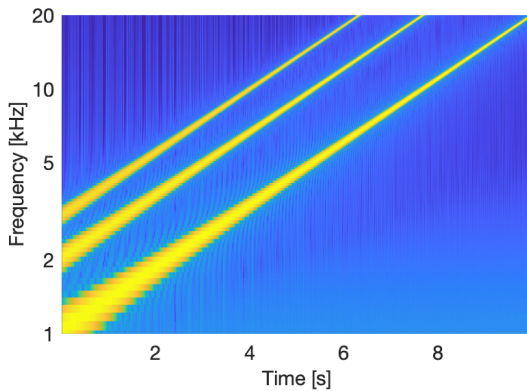
Question: what it sounds like ?



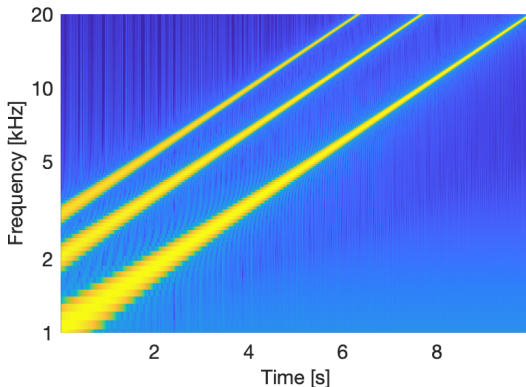
Answer 1: two independent impulses

Answer 2: impulse passed through a linear system
(direct sound and a reflection)

Question: what it sounds like ?



Question: what it sounds like ?

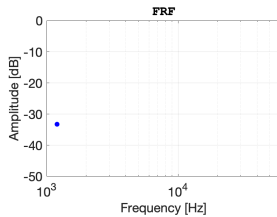
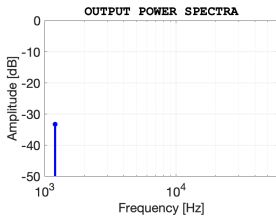
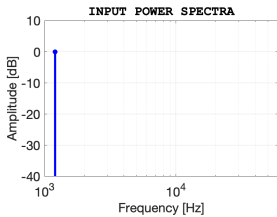
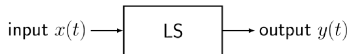
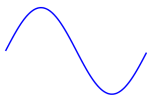


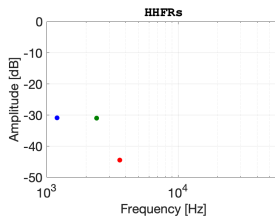
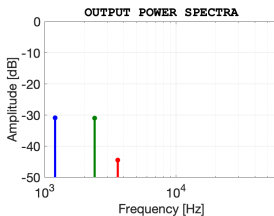
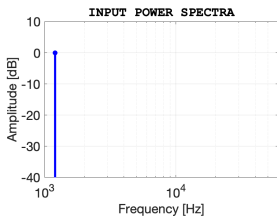
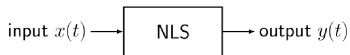
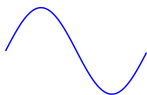
Answer 1: three independent swept-sines

Answer 2: swept-sine passed through a linear system
(direct sound and two reflections)

Answer 3: distorted swept-sine

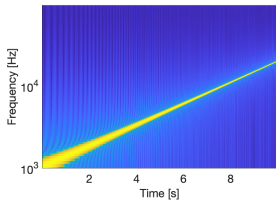
Higher Harmonic Frequency Response (HHFR)



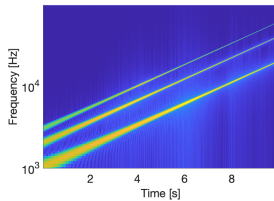


Synchronized Swept-Sine $\rightarrow$ HHFRs

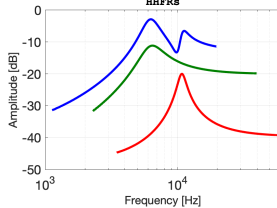
INPUT SPECTROGRAM



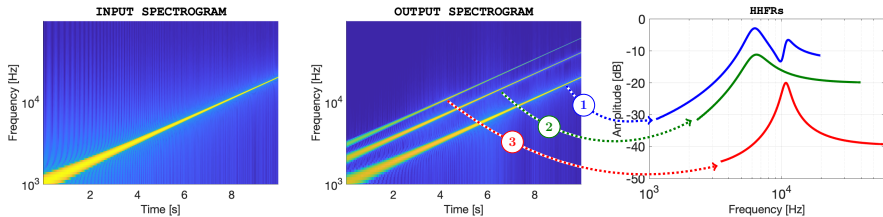
OUTPUT SPECTROGRAM



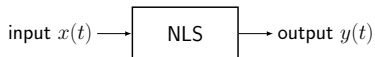
HHFRs



Synchronized Swept-Sine $\rightarrow$ HHFRs



... how it works ? ...

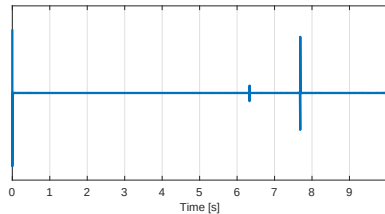


1) generate a swept-sine $x(t)$

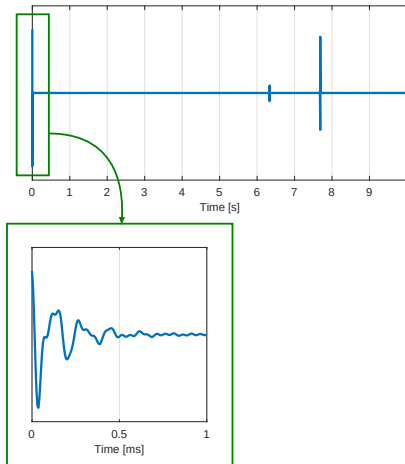
2) record $y(t)$

3) calculate FT $X(\omega)$, $Y(\omega)$ and $h(t) = \text{FT}^{-1} \left\{ \frac{Y(\omega)}{X(\omega)} \right\}$

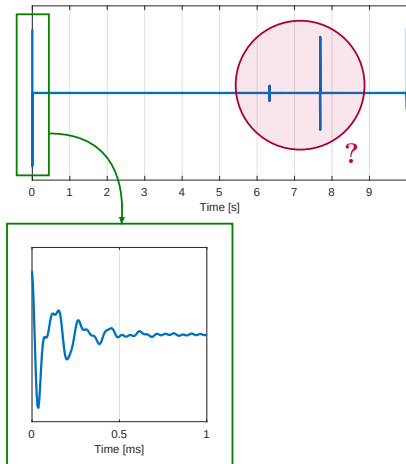
example of an impulse response $h(t)$



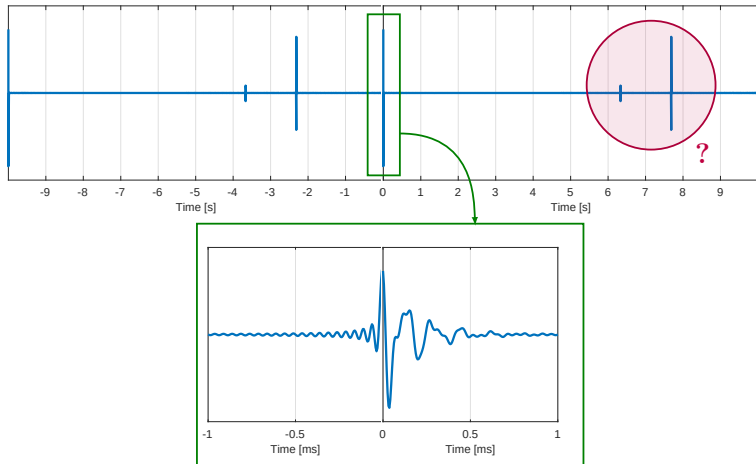
example of an impulse response $h(t)$



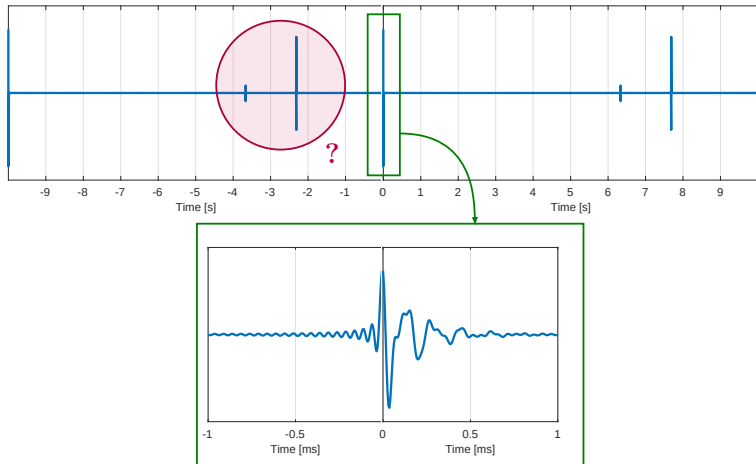
example of an impulse response $h(t)$



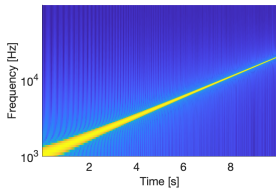
example of an impulse response $h(t)$



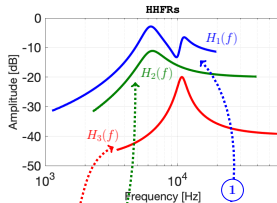
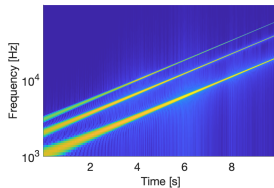
example of an impulse response $h(t)$



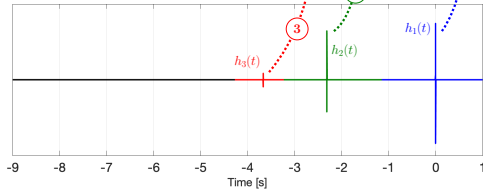
INPUT SPECTROGRAM

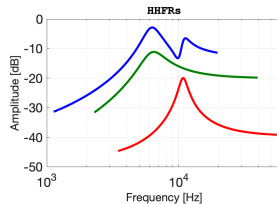
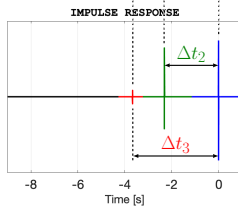
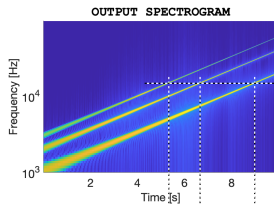
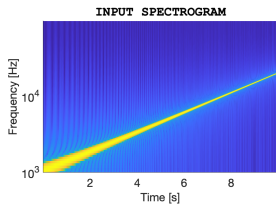


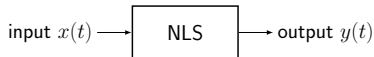
OUTPUT SPECTROGRAM



IMPULSE RESPONSE







1) generate a swept-sine $x(t)$

2) record $y(t)$

3) calculate FT $X(\omega)$, $Y(\omega)$ and $h(t) = \text{FT}^{-1} \left\{ \frac{Y(\omega)}{X(\omega)} \right\}$

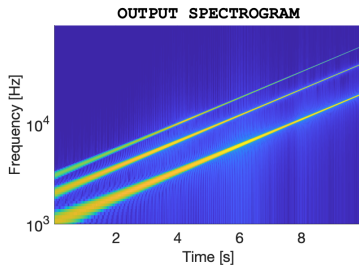
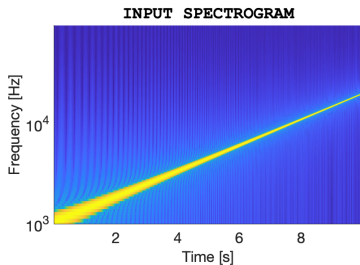
4) separate impulse response $h(t)$ to $h_m(t)$

5) calculate FT $H_m(\omega) \leftarrow \text{HHFRs}$

... once again ...

with some math

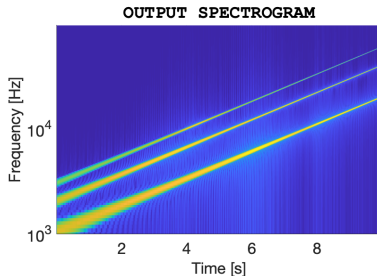
$$x(t) = \sin(\varphi(t)) \leftarrow \text{swept-sine}$$



$$\text{output} \rightarrow y(t) = \sin(\varphi(t)) + \sin(2\varphi(t)) + \sin(3\varphi(t))$$

output contains 3 harmonics (simple NLS)

$$y(t) = \sin(\varphi(t)) + \sin(2\varphi(t)) + \sin(3\varphi(t))$$



$$y(t) = \sin(\varphi(t)) + \sin(\varphi(t + \Delta t_2)) + \sin(\varphi(t + \Delta t_3))$$

$$\sin(\varphi(t)) = \sin(\varphi(t + \Delta t_1)), \quad \Delta t_1 = 0$$

$$\sin(2\varphi(t)) = \sin(\varphi(t + \Delta t_2))$$

$$\sin(3\varphi(t)) = \sin(\varphi(t + \Delta t_3))$$

we need such a phase $\varphi(t)$ that satisfies

$$\sin(m\varphi(t)) = \sin(\varphi(t + \Delta t_m))$$

$$m\varphi(t) = \varphi(t + \Delta t_m) + 2\pi l, \quad \forall l \in \mathbb{Z}$$

$$m\varphi(t) = \varphi(t + \Delta t_m) + 2\pi l, \quad \forall l \in \mathbb{Z}$$

is a Delayed Differential Equation

which has an exponential solution

$$\varphi(t) = A \exp(\alpha t)$$

with a little bit of more math

including boundary conditions for f_1 at $t = 0$ and f_2 at $t = T$

we can get

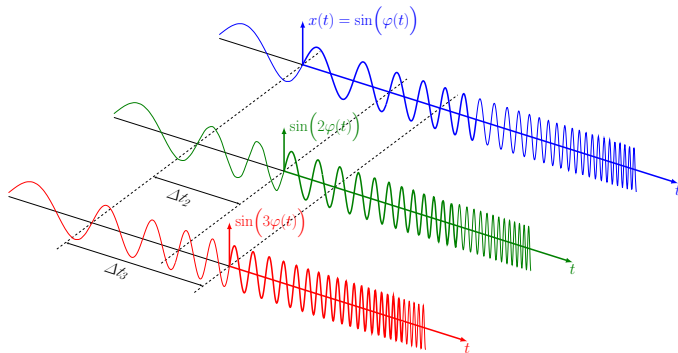
$$\varphi(t) = 2\pi f_1 L \exp\left(\frac{t}{L}\right)$$

$$\text{with } L = \frac{T}{\ln\left(\frac{f_2}{f_1}\right)}$$

Synchronized Swept-Sine

$$\varphi(t) = 2\pi f_1 L \exp\left(\frac{t}{L}\right)$$

$$\Delta t_m = L \ln(m)$$



1) signal generation

$$x(t) = \sin \left(2\pi f_1 L \exp \left(\frac{t}{L} \right) \right)$$

$$L = \frac{T}{\ln \left(\frac{f_2}{f_1} \right)}$$

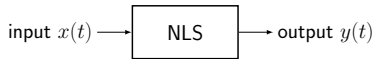
```
# -- swept-sine signal generation
```

```
f1 = 1e3           # start frequency
f2 = 20e3          # end frequency
T = 10             # time duration
```

```
L = T/np.log(f2/f1)
t = np.arange(0, T, 1/fs)
x = np.sin(2*np.pi*f1*L*np.exp(t/L))
```

```
# -- end of signal generation
```

2) record $y(t)$



record $y(t)$

as a response to swept-sine $x(t)$

3) calculate the Fourier Transform (FT) of $x(t)$ and $y(t)$ and find $h(t)$

$$X(\omega) = \text{FT}\{x(t)\}$$

$$Y(\omega) = \text{FT}\{y(t)\}$$

$$h(t) = \text{FT}^{-1}\left\{\frac{Y(\omega)}{X(\omega)}\right\}$$

```
## --Deconvolution
```

```
# FFT of the input signal  
X = np.fft.rfft(x)/fs
```

```
# FFT of the output signal  
Y = np.fft.rfft(y)/fs
```

```
# Frequency Response Function  
H = Y/X
```

```
# avoid Inf at DC  
H[0] = 0
```

```
# impulse response  
h = np.fft.irfft(H)
```

```
## -- end of Deconvolution
```

3) calculate the Fourier Transform (FT) of $x(t)$ and $y(t)$ and find $h(t)$

works, but can be done better

~~$$X(\omega) = \text{FT}\{x(t)\}$$~~

$$Y(\omega) = \text{FT}\{y(t)\}$$

$$h(t) = \text{FT}^{-1}\left\{\frac{Y(\omega)}{X(\omega)}\right\}$$

--Deconvolution

```
# FFT of the input signal  
X = np.fft.rfft(x)/fs
```

```
# FFT of the output signal  
Y = np.fft.rfft(y)/fs
```

```
# Frequency Response Function  
H = Y/X
```

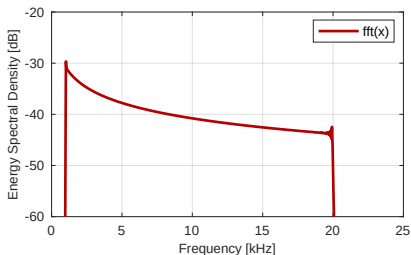
```
# avoid Inf at DC  
H[0] = 0
```

```
# impulse response  
h = np.fft.irfft(H)
```

-- end of Deconvolution

3) calculate the Fourier Transform (FT) of $x(t)$ and $y(t)$ and find $h(t)$

$$X(\omega) = \text{FT}\{x(t)\}$$

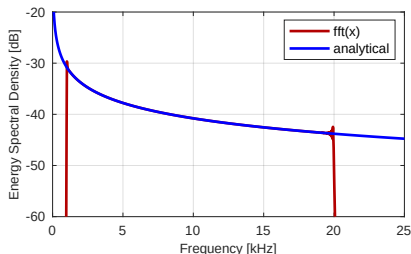


division by zeros problem

$$h(t) = \text{FT}^{-1} \left\{ \frac{Y(\omega)}{X(\omega)} \right\}$$

3) calculate the Fourier Transform (FT) of $x(t)$ and $y(t)$ and find $h(t)$

$$X(\omega) = \frac{1}{2} \sqrt{\frac{2\pi L}{\omega}} \exp \left\{ j\omega L \left[1 - \ln \left(\frac{\omega}{2\pi f_1} \right) \right] - j\frac{\pi}{4} \right\}$$



no division by zeros

$$h(t) = \text{FT}^{-1} \left\{ \frac{Y(\omega)}{X(\omega)} \right\}$$

```
# frequency axis
```

```
f_ax = np.fft.rfftfreq(len(y), 1/fs)
```

```
# analytical expression of the spectra of the synchronized swept-sine
```

```
# [Novak et al. (2015), JAES 63(10), pp.786-798., Eq. (42)]
```

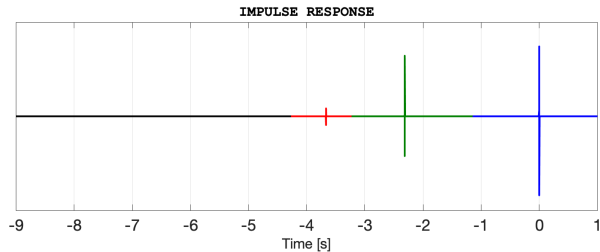
```
X = 1/2*np.sqrt(L/f_ax)*np.exp(1j*2*np.pi*
```

```
f_ax*L*(1-np.log(f_ax/f1))-1j*np.pi/4)
```

4) separate impulse response $h(t)$ to $h_m(t)$

$$h(t) = \sum_m h_m(t + \Delta t_m)$$

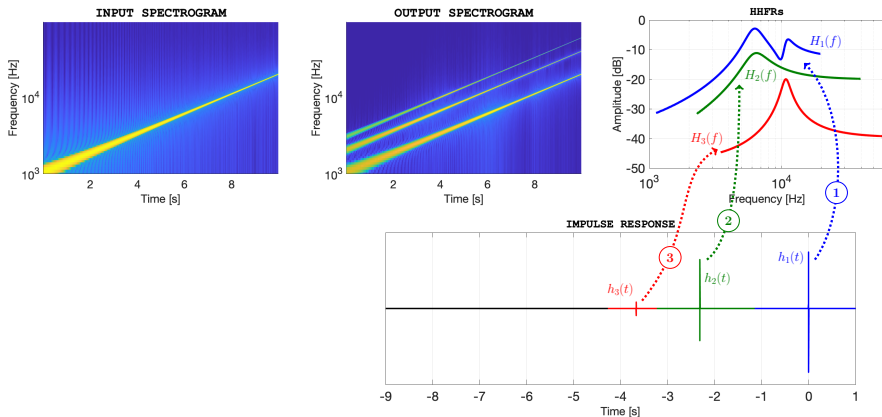
$$\Delta t_m = L \ln(m)$$



5) calculate FT $H_m(\omega) \leftarrow \text{HHFRs}$

$$H_m(\omega) = \text{FT}\{h_m(t)\}$$

% Higher Harmonic Frequency Responses
Hm = fft(hm);



References

<https://ant-novak.com/>

A. Novak, P. Lotton & L. Simon (2015), "Synchronized Swept-Sine: Theory, Application and Implementation", Journal of the Audio Engineering Society. Vol. 63(10), pp. 786-798.