



Digital Filtering

preparing for the exam - part 3, Z-plane & Crossovers

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Theoretical Examples

1. A digital filter is described by the transfer function

$$H(z) = \frac{b_0}{1 - 0.4z^{-1} + 0.29z^{-2}}.$$

- a Find the poles and zeros of the filter.
 - b Draw the poles and zeros into the unit circle.
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- (a) Find the poles and zeros of the filter.

$$H(z) = \frac{b_0 \cdot z^2}{z^2 - 0.4z + 0.29} \quad \leftarrow \text{double zero } z=0$$

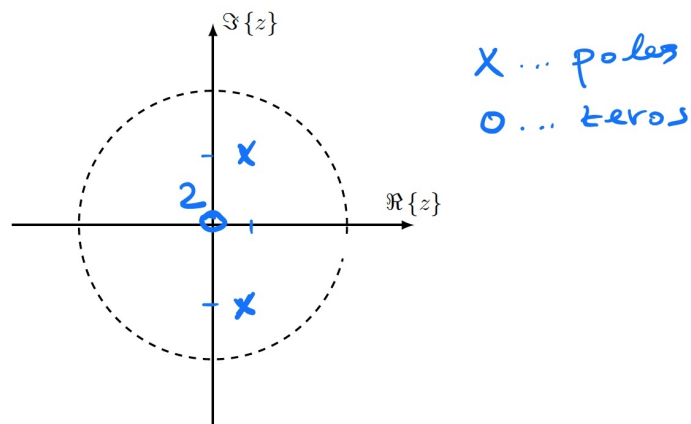
$$z^2 - 0.4z + 0.29 = 0$$

$$D = 0.16 - 4 \cdot 0.29 = -1$$

$$z_{1,2} = \frac{0.4 \pm \sqrt{-1}}{2} = 0.2 \pm j0.5$$

poles ↗

- (b) Draw the poles and zeros into the figure with the unit circle.



2. A 2nd order IIR filter has 2 poles $[0.3 + j0.4, 0.3 - j0.4]$ and two zeros $[0, -1]$. Find the filter coefficients **a** and **b**.
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A 2nd order IIR filter has 2 poles
 $0.3 + j0.4$, $0.3 - j0.4$ and two zeros
 0 , -1 .

Find filter coefficients a , b .

$$\begin{aligned} H(z) &= \frac{(z-0) \cdot (z+1)}{(z - (0.3 + j0.4))(z - (0.3 - j0.4))} = \\ &= \frac{z^2 + z}{z^2 - 0.3z + j0.4z - 0.3z - j0.4z + (0.3 - j0.4)(0.3 + j0.4)} = \\ &= \frac{z^2 + z}{z^2 - 0.6z + (0.09 + 0.16)} = \\ &= \frac{z^2 + z}{z^2 - 0.6z + 0.25} = \frac{1 + z^{-1}}{1 - 0.6z^{-1} + 0.25z^{-2}} \end{aligned}$$

coefficients: $b_0 = 1$, $b_1 = 1$
 $a_0 = 1$, $a_1 = -0.6$, $a_2 = 0.25$

Practical Examples

3. Prepare the filter coefficients for a 3-way 4th order Linkwitz-Riley (LR4) crossover in Python. The cutoff frequencies are $f_1 = 100$ Hz between the woofer and the midrange; $f_2 = 1$ kHz between the midrange and the tweeter; and the sample rate is 48 kHz. Export the coefficients of each filter in .txt files for Sigma Studio.

hint: `np.savetxt('coeffs.txt', np.append(b, a[1:]))`

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```
import scipy.signal as signal
import numpy as np

fs = 48e3
fc1 = 100
fc2 = 1e3

Wn1 = fc1/(fs/2)
Wn2 = fc2/(fs/2)

# low pass (2nd order IIR)
b1, a1 = signal.butter(2, Wn1, btype='low')

# band pass (two 2nd order IIR biquads in series)
b21, a21 = signal.butter(2, Wn1, btype='high')
b22, a22 = signal.butter(2, Wn2, btype='low')

# high pass (2nd order IIR)
b3, a3 = signal.butter(2, Wn2, btype='high')

# export for Sigma Studio
np.savetxt('coeffs_LP.txt', np.append(b1, a1[1:]))
np.savetxt('coeffs_BP1.txt', np.append(b21, a21[1:]))
np.savetxt('coeffs_BP2.txt', np.append(b22, a22[1:]))
np.savetxt('coeffs_HP.txt', np.append(b3, a3[1:]))

# each filter is applied twice
# woofer:    -- LP -- LP --
# midrange:  -- BP1 -- BP1 -- BP2 -- BP2 --
# tweeter:   -- HP -- HP --
```

4. Create a mono 3-way 4th order Linkwitz-Riley (LR4) crossover in Sigma Studio without using the block **Crossover**. You can use the block **General (2nd order)** that is under **Filters** - **Second Order** - **Double Precision** - **1 Ch**. The cutoff frequencies are $f_1 = 100$ Hz between the woofer and the midrange; $f_2 = 1$ kHz between the midrange and the tweeter.

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