

Modeling the response of a loudspeaker

We consider the loudspeaker with parameters :

- Electrical parameters
 - Coil resistance R_e [Ω]
 - Coil inductance L_e [H]
- Mechanical parameters
 - Mechanical compliance C_{ms} [m/N]
 - Mechanical moving mass M_{ms} [kg]
 - Mechanical resistance R_{ms} [kg/s]
- Coupling parameters
 - Coupling parameter $B\ell$ [T.m] [N/A]
 - Effective diaphragm area S_d [m²]

Fundamental equation of an electrodynamic loudspeaker

The fundamental equations are

$$u = R_e i + L_e \frac{di}{dt} + B\ell \frac{dx}{dt} \quad (1)$$

$$B\ell i = K_{ms} x + R_{ms} \frac{dx}{dt} + M_{ms} \frac{d^2 x}{dt^2}. \quad (2)$$

Laplace Domain analysis

In the Laplace domain the set of equation becomes

$$U(s) = (R_e + sL_e)I(s) + sB\ell X(s) \quad (3)$$

$$B\ell I(s) = (K_{ms} + sR_{ms} + s^2 M_{ms})X(s), \quad (4)$$

which, expressed using $Z_e(s) = R_e + sL_e$ and $Z_{ms}(s) = \frac{K_{ms}}{s} + R_{ms} + sM_{ms}$ leads to

$$U(s) = Z_e(s)I(s) + sB\ell X(s) \quad (5)$$

$$B\ell I(s) = sZ_{ms}(s)X(s). \quad (6)$$

Combining both previous equations we get

$$U(s) = s \left(\frac{Z_e(s) Z_{ms}(s)}{B\ell} + B\ell \right) X(s). \quad (7)$$

Taking into account the acceleration $A(s) = s^2 X(s)$ we can write

$$U(s) = s^{-1} \left(\frac{Z_e(s) Z_{ms}(s)}{B\ell} + B\ell \right) A(s). \quad (8)$$

which leads to

$$\frac{A(s)}{U(s)} = \frac{B\ell s}{Z_e(s) Z_{ms}(s) + (B\ell)^2}. \quad (9)$$

Putting back the expressions for impedances $Z_e(s)$ and $Z_{ms}(s)$, we can write

$$\frac{A(s)}{U(s)} = \frac{B\ell s}{(R_e + sL_e) \left(\frac{K_{ms}}{s} + R_{ms} + sM_{ms} \right) + (B\ell)^2}, \quad (10)$$

that can be written as

$$\frac{A(s)}{U(s)} = \frac{B\ell s^2}{(R_e + sL_e) (K_{ms} + s R_{ms} + s^2 M_{ms}) + s (B\ell)^2}. \quad (11)$$

Finally, the last equation can be written in fraction of polynomials

$$\frac{A(s)}{U(s)} = \frac{s^2}{s^3 A_3 + s^2 A_2 + s A_1 + A_0}, \quad (12)$$

with coefficients

$$A_0 = \frac{R_e K_{ms}}{B\ell} \quad (13)$$

$$A_1 = \frac{R_e R_{ms} + L_e K_{ms}}{B\ell} + B\ell \quad (14)$$

$$A_2 = \frac{R_e M_{ms} + L_e R_{ms}}{B\ell} \quad (15)$$

$$A_3 = \frac{L_e M_{ms}}{B\ell}. \quad (16)$$